Compensator Design for DC-DC Buck Converter using Frequency Domain Specifications

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Technology

in

Electrical Engineering

(Specialisation: Control & Automation**)**

by

GAURAV KAUSHIK

Department of Electrical Engineering

National Institute of Technology, Rourkela

2014

Compensator Design for DC-DC Buck Converter using Frequency Domain Specifications

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Technology

in

Electrical Engineering

(Specialisation: Control & Automation)

by

GAURAV KAUSHIK

ROLL NO: 212EE3230

Under the Guidance of

PROF. SUSOVON SAMANTA

Department of Electrical Engineering

National Institute of Technology, Rourkela

2014

National Institute Of Technology
Rourkela

CERTIFICATE

This is to certify that the thesis entitled, ― Compensator Design for DC-DC Buck Converter using Frequency Domain Specifications "submitted by Mr Gaurav Kaushik in partial fulfilment of the requirements for the award of Master of Technology Degree in Electrical Engineering with specialization in "CONTROL AND AUTOMATION" at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date:

Dr. SUSOVON SAMANTA
Department of Electrical Engineering
National Institute of Technology

ACKNOWLEDGEMENTS

I am very thankful to my guide Prof. Susovon Samanta for providing me his precious knowledge and showed trust in me during the hard time. His constant advice and efforts kept me going during my whole research work. His deep knowledge and command over the subjects helped me to learn a lot and had grown an insight in me. He has helped me a lot and provided me all his knowledge to accomplish this great task.

I would also like to mention our head of department Prof.A.K. Panda to provide me with best of the resources and facilities at NIT Rourkela. I am also thankful to Prof. Sandip Ghosh, Prof. Subojit Ghosh, and Prof. B.D.Subudhi for their help and blessings.

Last but not the least I would also like to thank all my friends and batch mates to always encourage and motivate me to work hard. They helped me during my hard times and helped me to face that patiently.

Gaurav Kaushik

Contents

LIST OF FIGURES:

Abstract:

In recent times integrated power management circuits have emerged as an important component of the portable application market. Designing a power supply for meeting high efficiency and good transient response has been a major topic for research in recent years. The DC- DC converter demands are increasing due to their small size, high efficiency and easy to use characteristics. In this study, we have studied few ways to design the controllers for the DC-DC converter which can control the ripple content of the system to achieve the required performance and good regulated voltage. The methods described can be implemented in hardware circuits very easily. The frequency domain specifications are used to tune the controllers as they have more effect on their performance and the calculations get simpler when working in frequency domain. These methods gives the exact values that can be directly used unlike the earlier used trial and error procedures. They are designed for the voltage mode controlled buck converter topology. Various controllers like PID, TYPE-III Controller and hardware simulation is done to verify the result.

Chapter 1

Introduction

Background

Dc-Dc converters are the converters that are used to convert one voltage level to another voltage that may be higher in the magnitude or lower in the magnitude. These Dc-Dc converters are used everywhere because of their high efficiency and single stage conversion. The control of voltage is done by controlling the duty ratio of the switch. Switches used are Mosfets, transistors, GTO's, IGBT's depending upon the circuit or the power transfer capability. Due to recent hike in the demand of portable devices like mobiles, laptops and use of regulated power supplies in the aerospace application, in automotive industries. In these systems the load voltage is kept constant irrespective of the load and supply [1]. Dc-Dc converters are used extensively because in AC system you can convert the voltage levels by the use of transformer but in DC system case is different so these are essential for change in voltage levels in dc system. The reason for their increased use is their cost effectiveness and simple circuitry. There is no energy generated inside the converter, all the energy that is supplied by source is transferred to load with little losses, to different voltage and current level. The applications where they are used day to day is running of CD player, to supply the motherboard of personal computers. They are also used in the satellites where dc buses at different voltage levels are supplied through these dc-dc converters.

They are of two kinds:

1. Non Isolated Converters
2. Isolated Converters

Non Isolated Converters:

In these type of converters the voltage level step-up or step-down ratio is not that much high to create a problem and can be used without isolation. [2]The topologies that are generally used for this category are buck, boost, buck-boost and Cuk. They share common connection.

Isolated Converters:

In these type of converters the voltage level step-up or step down ratio is very high so that use of electrical isolation is indispensable. Here output side is completely isolated from the source side. This ensures the safe operation of converter. There are two topologies that are used largely in this category are flyback converter and forward converter. For us the concern is the non-isolated dc-dc converter.

Buck Converter:

In this converter the output is connected to the source during the time switch is on and when switch is off output is supplied through the capacitor and inductor via freewheeling diode [2]. In this way the output current is continuous and load voltage switches between the Vin and zero. The average load voltage is less than the supplied input voltage.

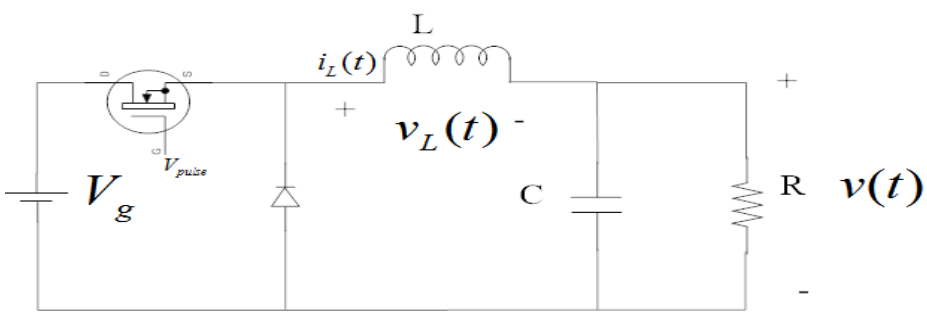


Figure 1 Buck Converter

Boost Converter:

[2]In this converter the output voltage is more than the supply voltage. When switch is on source charges the inductor and inductor stores energy, when switch is off the load is supplied by source

through inductor. The voltage level is boosted as the inductor supplies its stored energy to the load in second half.

When the above described converters operate in the open loop configuration the ripple in the output voltage is very high and this very dangerous if the output is given to IC's as there tolerance range is quite small. So for that we need to control the duty cycle of the converter according to our requirements. There are two ways of controlling duty cycle of converters

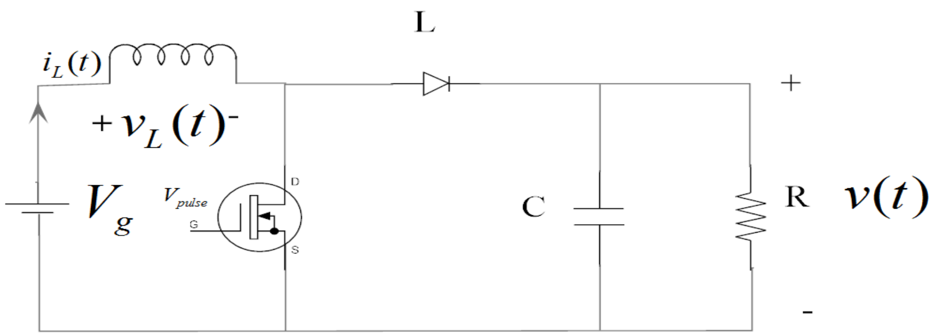


Figure 2 : Boost Converter

1. Voltage Mode Control
2. Current mode Control

Voltage Mode Control:

Here voltage is sensed by sensor and then compared to required output voltage and then it is used to generate PWM pulses to drive Mosfet. [3]This variations may increase/decrease the duty ratio of Mosfet. Compensator is employed here so that the variations become small and high switching may not burn the mosfet.

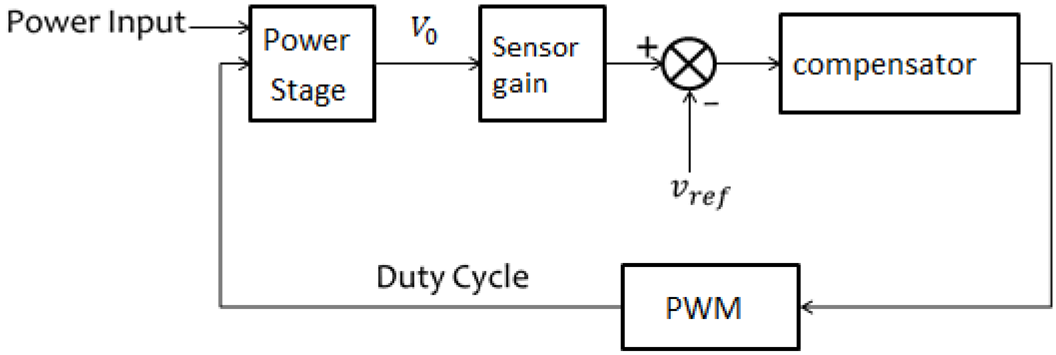


Figure 3: Voltage Mode Control

Current Mode Control:

In this mode the current input to inductor is sensed by the sensor and then it is compared with the controllers output and fed to the SR flip flop so that the pulses [4]can be generated to drive the Mosfet. It has two loops one inner current loop that controls the inductor current and outer voltage loop that controls output voltage which in turn is controlled by the inner loop.

But it has some advantage over the voltage mode control:

- Current through the switch is limited to its maximum value so that the switch do not get burnt or damaged.
- Protection during overload.
- Operating the converters in parallel is easy with this control.

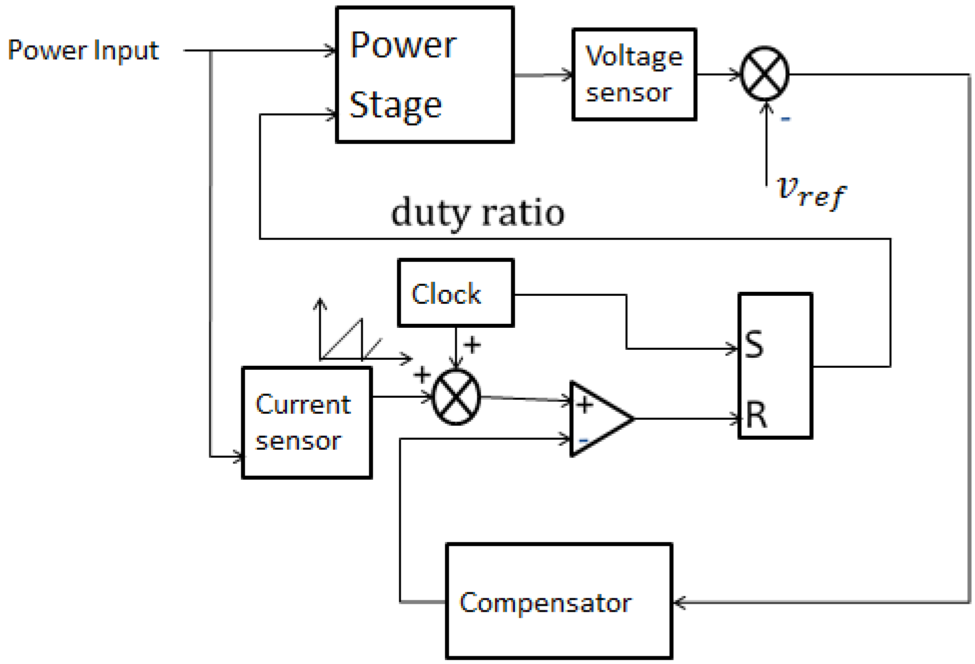


Figure 4: Current Mode Control

1.2 Motivation:

As the need of the portable devices are increasing day by day and the need for cheap and efficient regulated power supplies are increasing simultaneously. The need for controlled ripple regulated supply makes it indispensable to search for the compensator design such that closed loop control of the voltage make the ripple in the range of 1-2% of supply voltage.

1.3 Organisation of Thesis:

This thesis work is divided into five chapters. Chapter 1 gives a brief introduction about the DC-DC converters and their background. This tell us what the different types of configuration are and the how one is different from another and what their individual advantages are. Chapter 2 describes how we model the buck regulator and derive the transfer function to be used for designing the compensator. The small signal analysis is done to see the variation of the converter characteristics around the desired point of operation. Chapter 3 covers how to select the compensator and make correct choice while there is confusion between any compensators. Various compensators are shown and there effects on the system are described so as to ease the process of selection. Chapter 4 describes few methods taken from literature to design the PID controller, Type-II compensator, Type III compensator and their frequency responses are shown. The hardware design and their responses are also shown. Chapter 5 describes the conclusions drawn from the work done and suggested future work that can be done in this area.

Chapter 2

Chapter 2

Small Signal Analysis of Buck Converter

2.1 Introduction

Small signal analysis is done to know the dynamics of the system and design the compensators for the switching converters. The small signal models include various transfer functions such as control to output, output impedance, audio susceptibility etc. Therefore we can design the compensator according to our choice regarding any of these characteristic of transfer function. The main purpose of doing small signal analysis is to see the ac behavior of the switching converter around a fixed operating point.

There are various methods [2]that model these time variant systems into linear time invariant systems. State space averaging, Circuit averaging, Current injected approach are some of them. For our our analysis we will take into account only state space averaging technique.

2.2 State Space Description for Each Interval

Here it is assumed that the buck converter is in continuous conduction mode. Therefore two circuits are considered, one for the on time and other for the off time of the converter. During T_{on} the switch is on and supply is connected to load through the inductor as shown.

So for on time our equation for inductor voltage and capacitor current are,

$$\boldsymbol{v_l(t) = v_g(t) - v_c(t)} \ldots\ldots\ (1)$$

$$\boldsymbol{i_c(t) = i_l(t) - i_o(t)} \ldots\ldots\ (2)$$

By expanding above equation we get,

$$\boldsymbol{L\frac{di_l}{dt} = v_g(t) - v_c(t)} \ldots\ldots\ (3)$$

$$\boldsymbol{\frac{di_l}{dt} = \frac{v_g(t) - v_c(t)}{L}} \ldots\ldots\ldots\ldots\ (4)$$

Similarly for the capacitor current,

$$\boldsymbol{C\frac{dv_c}{dt} = i_l(t) - i_o(t)}\text{.....}\ (5)$$

$$\boldsymbol{\frac{dv_c}{dt} = \frac{i_l(t)}{C} - \frac{v_c(t)}{RC}}\text{.........}\ (6)$$

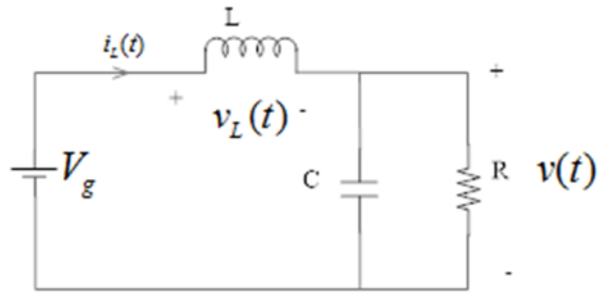


Figure 5: During On Time

Now the above equations can be written as,

$$\begin{bmatrix} \frac{di_l}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ -1/C & -1/RC \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} -1/L \\ 0 \end{bmatrix} [v_g]$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix}$$

They may be written as,

$$\dot{x} = A_{on}x + B_{on}u$$

$$y = C_{on}x + D_{on}u$$

Here, D_{on}=0

Now we analyze the off time circuit,

During off time the switch is open and the load current is supplied by the inductor stored energy.

And this path is completed through the diode.

During this time inductor voltage is,

$$\boldsymbol{v_l(t) = -v_c(t)}$$

$$\boldsymbol{\frac{di_l}{dt} = -\frac{v_c(t)}{L}}$$

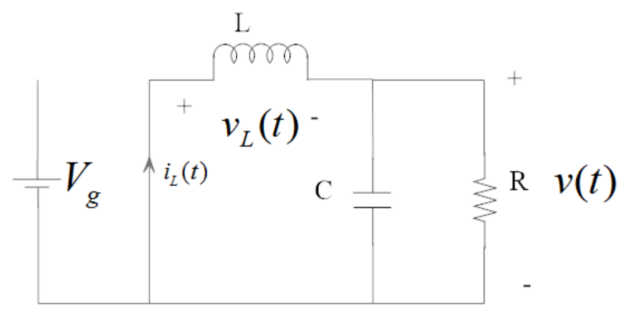


Figure 6: During Off Time

Similarly for capacitor current,

$$i_c(t) = i_l(t) - i_o(t)$$

$$\frac{dv_c}{dt} = \frac{i_l(t)}{C} - \frac{v_c(t)}{RC}$$

These can also be written as,

$$\begin{bmatrix} \frac{di_l}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1/C & -1/RC \end{bmatrix} \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [v_g]$$

$$y = [0 \quad 1] \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix}$$

Above equations can be rewritten as,

$$\dot{x} = A_{off}x + B_{off}u$$

$$y = C_{off}x + D_{off}u$$

Here $D_{off} = 0$

From above equations we can see that $A_{on} = A_{off}$ and $B_{off} = 0$

2.3 State Space Averaging

Let the converter switch be on for the time d_nT i.e. t_{on} and d'_nT is the interval for which switch is off i.e. $t_{off} = d'_nT = (1 - d_n)T$.

The state space description can be written as

ON: $\dot{x}(t) = A_{on}x(t) + B_{on}u(t)$ for time $t \in [nT, nT + d_nT]$, n=1, 2, 3… (1)

OFF:$\dot{x}(t) = A_{off}x(t) + B_{off}u(t)$ for time $t \in [nT + d_nT, (n+1)T\ ]$ n=1, 2, 3… (2)

The solutions to the above equations can be found by taking the integration over each interval of operation,

$$x(n + d_n)T = e^{A_{on}d_nt}x(nT) + A_{on}^{-1}(e^{A_{on}d_nt} - I)B_{on}u \tag{3}$$

$$x(n + 1)T = e^{A_{off}d'_nT}x(nT + d_nT) + A_{off}^{-1}(e^{A_{off}d'_nT} - I)B_{off}u \tag{4}$$

Substituting (3) in (4)

$$x(n + 1)T = \left(e^{A_{off}d'_nT}(e^{A_{on}d_nt}x(nT) + A_{on}^{-1}(e^{A_{on}d_nt} - I)B_{on}u)\right) + A_{off}^{-1}(e^{A_{off}d'_nT} - I)B_{off}u$$

$$= e^{A_{off}d'_nT}e^{A_{on}d_nt}x(nT) + A_{on}^{-1}\left(e^{A_{off}d'_nT}e^{A_{on}d_nt} - e^{A_{off}d'_nT}\right)B_{on}u +$$

$$A_{off}^{-1}(e^{A_{off}d'_nT} - I)B_{off}u \quad (5)$$

Then we get

$$x(n+1)T = e^{(A_{on}d_n + A_{off}d'_n)T}x(nT) + A_{on}^{-1}\left(e^{A_{off}d'_nT}e^{A_{on}d_nt} - e^{A_{off}d'_nT}\right)B_{on}u +$$

$$A_{off}^{-1}\left(e^{A_{off}d'_nT} - I\right)B_{off} \quad (6)$$

Now introducing the linear ripple approximation,

$$e^{AT} = I + AT$$

Applying this we can get,

$$x(n+1)T = x(nT) + \left(A_{on}d_n + A_{off}d'_n\right)Tx(nT) + d_nTB_{on}u + d'_nTB_{off}u \quad (7)$$

Expanding through Euler's approximation we can approximate derivatives as follows,

$$\dot{x} = \frac{x(nT+T) - x(nT)}{T}$$

Applying this to the equation (7) we get,

$$\dot{x} = \left(A_{on}d_n + A_{off}d'_n\right)x + (B_{on}d_n + B_{off}d'_n)u \quad (8)$$

$$y = \left(C_{on}d_n + C_{off}d'_n\right)x + (E_{on}d_n + E_{off}d'_n)u \quad (9)$$

Intrinsically the d is a discrete quantity with single value over a cycle. Therefore $d_n(t)$ can be replaced by *d(t)* assuming a very small variations occur that can be neglected.

$$\dot{x} = \left(A_{on}d(t) + A_{off}d'(t)\right)x + (B_{on}d(t) + B_{off}d'(t))u \quad (10)$$

$$y = \left(C_{on}d(t) + C_{off}d'(t)\right)x + (E_{on}d(t) + E_{off}d'(t)) \quad (11)$$

2.4 Linearization:

The equations derived above are non-linear and we have to linearize them. [1]To linearize and obtain small signal model we have to perturb them around an operating point (D, X, U). Let,

$$d = D + \hat{d}, x = X + \hat{x}, u = U + \hat{u} \tag{11}$$

For small signal model we consider only first order terms and we obtain,

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} + \left(\left(A_{on} - A_{off}\right)X + (B_{on} - B_{off})U\right)\hat{d} \tag{12}$$

$$\hat{y} = C\hat{x} + E\hat{u} \tag{13}$$

Where $A = DA_{on} + D'A_{off}$,$B = DB_{on} + D'B_{0ff}$,$C = DC_{on} + D'C_{off}$,$E = DE_{on} + D'E_{off}$

The steady state value of duty ratio and state variables are obtained by considering the constant terms equal to zero and given as:

$$AX + BU = 0 \tag{14}$$

$$Y = CX + EU \tag{15}$$

So using equation (14) & (15) we get,

$$Y = -CA^{-1}BU + EU \tag{16}$$

Now extracting the small signal model:

Applying Laplace transform to the equation and considering initial conditions to zero we get,

$$s\hat{x}(s) = A\hat{x}(s) + B\hat{u}(s) + ((A_1 - A_2)X + (B_1 - B_2)U)\hat{d} \tag{17}$$

$$(sI - A)\hat{x}(s) = B\hat{u}(s) + ((A_1 - A_2)X + (B_1 - B_2)U)\hat{d} \tag{18}$$

$$\hat{x}(s) = (sI - A)^{-1}\left[B\hat{u}(s) + ((A_1 - A_2)X + (B_1 - B_2)U)\hat{d}\right] \tag{19}$$

By solving for above equation we get control to output transfer function we get,

$$G_{vd} = \frac{\widehat{v_0}}{\hat{d}} = \frac{RV_g}{(s^2RLC+sL+R)} \tag{20}$$

If all the non-ideal case is considered, then during on time the converter circuit will be like this.

The corresponding state space equation,

$$\begin{bmatrix} \dot{i_L} \\ \dot{v_c} \end{bmatrix} = \begin{bmatrix} -\frac{1}{L}(r_L + r_{dson} + \frac{Rr_c}{R+r_c} & -\frac{1}{L}\frac{R}{(R+r_c)} \\ \frac{R}{C(R+r_c)} & -\frac{1}{C(R+r_c)} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & -\frac{r_c R}{L(R+r_c)} & 0 \\ 0 & \frac{R}{C(R+r_c)} & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} v_0 \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{Rr_c}{R+r_c} & \frac{R}{R+r_c} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & \frac{r_c R}{R+r_c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \tag{22}$$

When the switch is off the circuit of converter shown below

$$\begin{bmatrix} \dot{i_L} \\ \dot{v_c} \end{bmatrix} = \begin{bmatrix} -\frac{1}{L}(r_L + r_d + \frac{Rr_c}{R+r_c} & -\frac{1}{L}\frac{R}{(R+r_c)} \\ \frac{R}{C(R+r_c)} & \frac{1}{C(R+r_c)} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & -\frac{r_c R}{L(R+r_c)} & -\frac{1}{L} \\ 0 & \frac{R}{C(R+r_c)} & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} v_0 \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{Rr_c}{R+r_c} & \frac{R}{R+r_c} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & \frac{r_c R}{R+r_c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \tag{24}$$

By using state space averaging as done in above ideal case and after linearizing the equation it can be write that,

$$V_0 = \frac{Rr_c}{R+r_c} I_L + \frac{R}{R+r_c} V_c \tag{25}$$

Where, $I_L = \frac{D_p V_g}{a} - \frac{(1-D_p)V_d}{a}$ and $V_c = \frac{RD_p V_g}{a} - \frac{R(1-D_p)V_d}{a}$ (26)

By putting the value of I_L and V_c in equation no. (5)

$$V_0 = \frac{Rr_c}{R+r_c} \times \frac{D_p V_g}{a} - \frac{Rr_c}{R+r_c} \times \frac{(1-D_p)V_d}{a} + \frac{R}{R+r_c} \times \frac{RD_p V_g}{a} - \frac{R}{R+r_c} \times \frac{R(1-D_p)V_d}{a} \tag{27}$$

Now, $r_{on} = r_L + r_{dson}$, $r_{off} = r_L + r_d$ and $a = R + D_P r_{on} + D_P r_{off}$

So, for a given output voltage and input voltage the duty ratio with parasitic is derived as,

$$D_p = \frac{V_0(R+r_{off})+V_d R}{V_0(r_{off}-r_{on})+R(V_g+V_d)} \tag{28}$$

After obtaining steady state values of D_P , I_L and V_c, all the transfer function arising out of the state space model can be derived for open loop power stage. Assume that the state of the incremental linearizing model is zero initially.

Then the transfer function is,

$$G_{vd}(s) = \frac{R(V_g+V_d+I_L(r_{off}-r_{on})(1+sCr_c}{\Delta(s)} \tag{29}$$

Where,

$$\Delta(s) = s^2 LC(R + r_c) + s\left[L + RC\left\{\left(r_c + r_{off}\right) + D_p\left(r_{on} - r_{off}\right)\right\} + r_c C\left\{D_p\left(r_{on} - r_{off}\right) + r_{off}\right\}\right] + R + D_p r_{on} + D_p r_{off} \tag{30}$$

Chapter 3

Chapter 3

Compensator Design for DC-DC Converter

Fixed structure compensators are those compensators whose design remain fixed for a particular DC-DC converter and for a change in plant only values of the resistance and capacitance values need to be changed. There are many ways to design these compensators based on the time-domain and frequency-domain specification as these characteristics define the characteristics of the system. Here we will show how to design the compensator based on the frequency-domain specification.

3.1 Frequency-Domain Specifications [5]:

1. Gain Margin: it is the amount of change required in open loop gain to make the system unstable.
2. Phase Margin: it is the amount of change required in open loop phase shift to make a closed loop system unstable.
3. Bandwidth: the bandwidth is frequency at which the closed loop systems gain falls to -3db.
4. Nyquist Slope: its new and effective method to shape the loop characteristics.

3.2 Effects of Frequency Domain Specifications on the System:

1. Gain Margin: increasing gain margin helps us to remove low frequency noise problems and increase in dc gain also makes system a little faster.
2. Phase margin: low phase margin causes closed loop system to exhibit overshoot and ringing. It is closely related to closed loop damping factor.

3. Bandwidth: it helps to remove the sensor noise used to sense the current/voltage in the closed loop system. It also affect the rise time of the system.
4. Nyquist slope: it helps to shape the loop so that the slope of magnitude curve is -20db/decade at crossover frequency and the phase curve gives phase margin of the -90 so that robust controllers may be designed.

3.3 Selection of compensator [3]:

There are four compensators among which we have to choose P, PI, PD and PID. The selection is based on the following characteristics:

a. Rise Time
b. Maximum Overshoot
c. Steady State Error
d. Settling Time

We have to select what suits best to our requirement.

3.4 Proportional controller:

It is mainly used to reduce the steady state error and as the gain of proportional controller is increased the steady state error starts to decrease with decrease in rise time. But despite the reduction in the steady state error it can never completely eliminate the error. Also it increases the maximum overshoot of the system. It makes the dynamics of the system faster with larger bandwidth but also makes system more susceptible to noise. Large value of K (proportional gain) can also make system unstable.

3.5 Proportional plus Integral (PI):

Integral term is mainly used to eliminate the steady state error but at the cost of the reduced speed of the system. Proportional term used is used to compensate the lag produced by integral term but overall system lags its earlier speed. Also it can boost the oscillations and maximum overshoot.

3.6 Proportional plus Derivative (PD):

It is mainly used to boost the dynamics of the system and increase the stability of the system. The derivative control is not used alone because of the problem of amplifying the noise.it has very less effect on the steady state error of the system.

3.7 Proportional plus Integral plus Derivative (PID):

PID has fast dynamics, zero steady state error and no oscillations with high stability. It has the advantage is that it can be applied to system of any order. Derivative gain in addition to PI is to increase the speed of response and decrease overshoot and oscillations. There are lots of tuning methods available including online tuning including iterative procedures, offline tuning using the pen and pencil. According to the advantages offered by PID over other we select PID and here we present two methods to tune the parameters of PID.

Chapter 4

Chapter 4

Compensator Tuning Methods

4.1 Methods to Tune PID:

Two methods are as follows:

1) PID controller tuning using Bode's Integral
2) Exact tuning of PID controller based on the PM, Bandwidth and GM

4.2 PID controller tuning using Bode's Integral:

Motivation:

This method to tune is selected because this method can give the vales of controller parameters in single step and we don't have to go for the iterative method and also this method provides with the equation that can be with pen and paper instead of going for any software [3].

This method uses the two new relations derived from Bode's integral i.e. derivative of amplitudes relation to phase of the system and the derivative of phase relation to amplitude of the system at the crossover frequency. Also it uses one term called the Nyquist slope of the curve that will be used to shape the loop response around the crossover frequency. These two relations are as follows:

$$s_a(\omega_c) = \omega_c \times \frac{d \ln|G(j\omega_c)|}{d\omega} \approx \frac{2}{\pi} \angle G(j\omega_c) \quad \text{.... (1)}$$

$$s_p(\omega_c) \approx \angle G(j\omega_c) + \frac{2}{\pi}\left[\ln|k_g| - \ln|G(j\omega_c)|\right] \quad \text{..... (2)}$$

Where $s_a(\omega_c)$ = derivative of amplitudes relation to phase of the system

$s_p(\omega_c)$ = derivative of phase relation to amplitude of the system

ω_c = crossover frequency

k_g = system static gain

$G(j\omega_c)$ = is the plant on which we have to apply controller

Tuning steps:

1. The parallel form of PID:

$$k(j\omega) = k_p \left(1 + \frac{1}{j\omega T_i} + j\omega T_d\right) \quad \text{..... (3)}$$

2. Select the phase margin (ϕ_d) and the desired crossover frequency (ω_c).
3. Select the desired slope of the Nyquist curve (ψ).
4. Now calculate the values of the following:

$$s_p(\omega_c) = \omega \frac{d\angle G(j\omega)}{d\omega}\bigg|_{\omega=\omega_c} \quad \text{..... (4)}$$

$$s_a(\omega_c) = \omega \frac{d \ln |G(j\omega)|}{d\omega}\bigg|_{\omega=\omega_c} \quad \text{...... (5)}$$

5. Now we can calculate the parameters of the PID:

$$k_p = \frac{|\cos(\phi_d - \phi_c)|}{|G(j\omega_c)|} \quad \text{....... (6)}$$

$$T_i = \frac{1}{\omega_c (T_d \omega_c - \tan(\phi_d - \phi_c))} \quad \text{...... (7)}$$

$$T_d = \frac{1}{2\omega_c}[(s_a(\omega_c) - s_p(\omega_c)\tan(\phi_d - \phi_c))\tan(\psi - \phi_c) + (1 - s_a(\omega_c))\tan^{-1}(\phi_d - \phi_c) - s_p(\omega_c)] \quad \text{..(8)}$$

Note:

If the plant has pole at origin then the systems static gain is calculated without that pole and then applied to equation(2) to get the value of derivative of phase.

4.3 Verification of the Method:

The above is applied to the buck converter to verify whether the parameter values obtained from the previous method gives the satisfactory result or not.

Parameter values of the Buck converter:

Inductor (L) =50µH, Output Capacitor(C) =500µF, R=3Ω, Switching Frequency=100 kHz

Input voltage=28V, Output voltage=15V, Output current=3A

For the above specifications, the control to output transfer function is as follows,

$$G(s) = \frac{v_0}{d} = \frac{2.33}{s^2 \times 2.58 \times 10^{-8} + s \times 16.67 \times 10^{-6} + 1}$$

We choose $\emptyset_d$=52^0 ,$f_c = 5kHz$ and ψ=35^0, and we get controller parameters as follows,

$$k_p = 6.42, T_i = 1.35 \times 10^{-3}, T_d = 40 \times 10^{-6}$$

4.3.1 Response of the System with PID:

Closed loop model of the system:

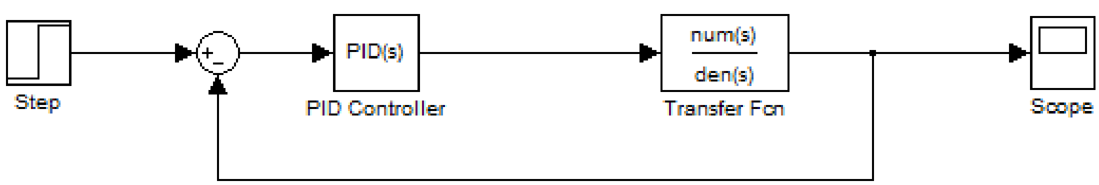


Figure 7: Closed loop model of the system

Simulink Model of Plant:

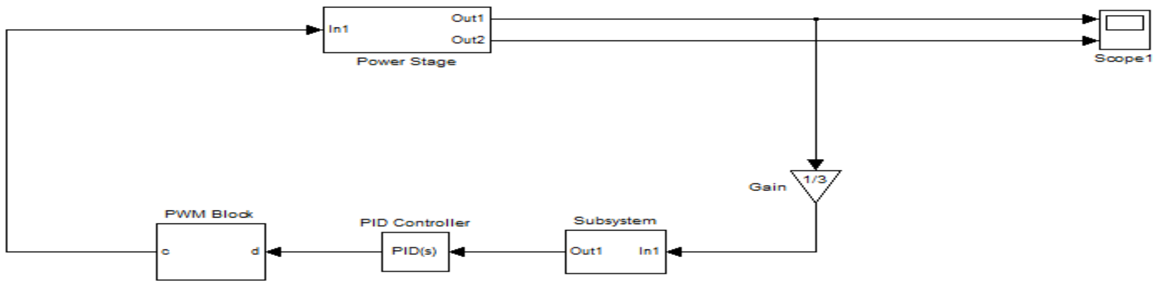


Figure 8: Simulink Model of Plant

Step Response of System:

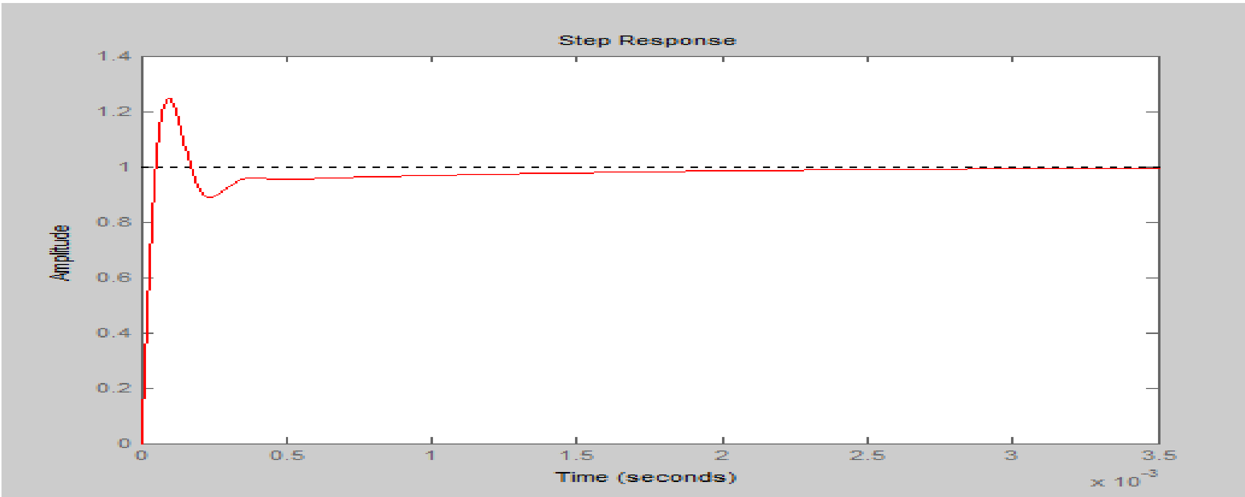


Figure 9: Step Response of System

Bode Plot of the System with PID:

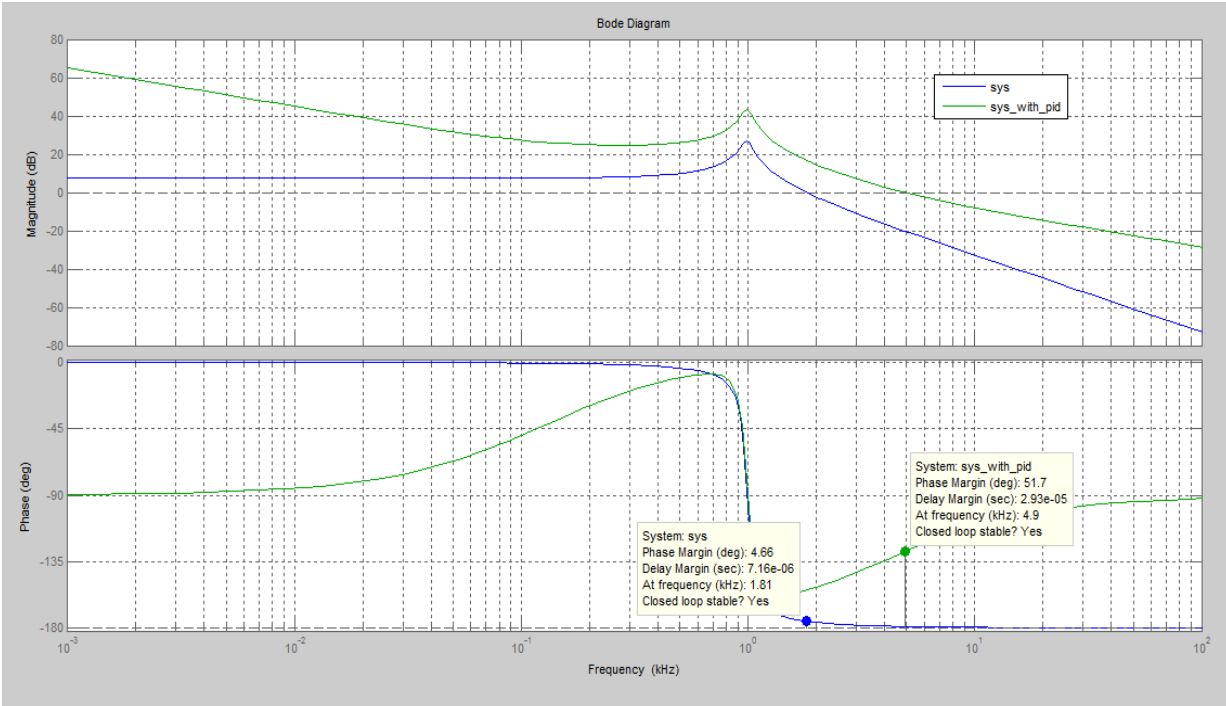


Figure 10: Bode Plot of the System with PID

Inference from plot:

- The dc gain of the system with PID is very large as compared to the original system. This helps to remove the low frequency noise and also improves the rise time of the system.
- We can also see that the bandwidth is also increased which is done intentionally to improve the systems immunity against sensor noise and against the high frequency noise.

- We see that our desired phase margin is achieved to large extent and also we see that in high frequency region our phase is always 90^0 which means that our controller is robust in terms that at any frequency above bandwidth our phase will remain 90^0.

Load Disturbance Rejection of Controller:

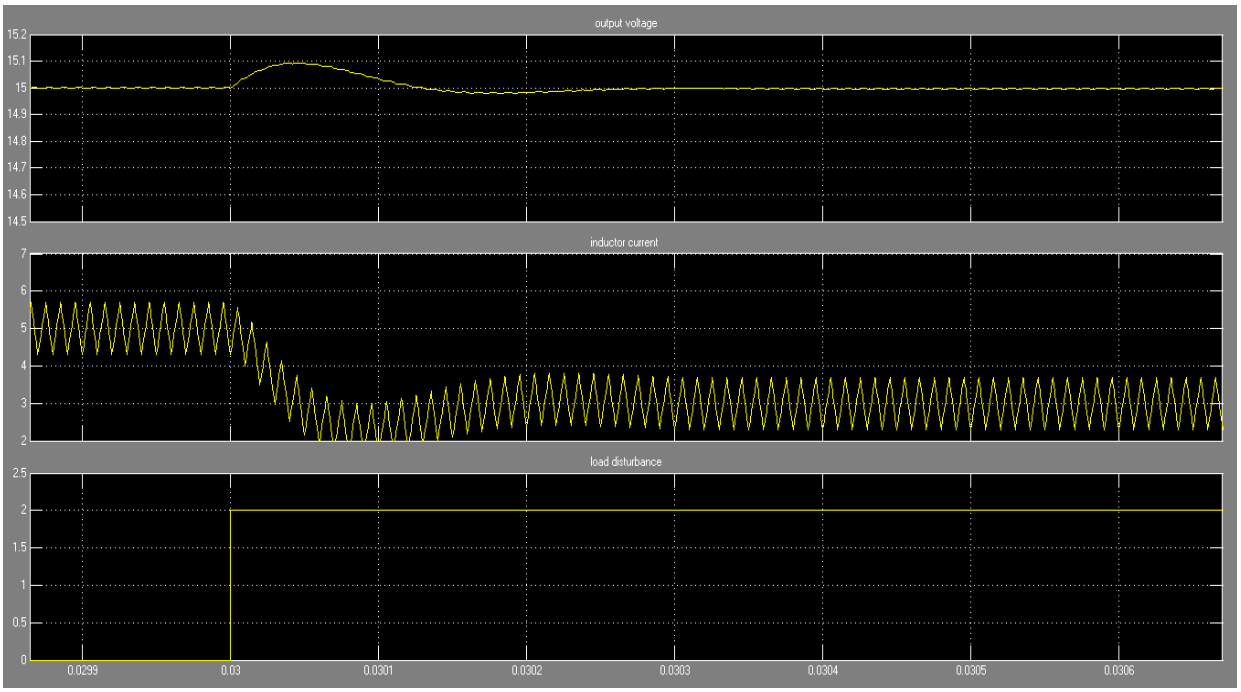


Figure 11: Load Disturbance Rejection of Controller

Output Reference Tracking:

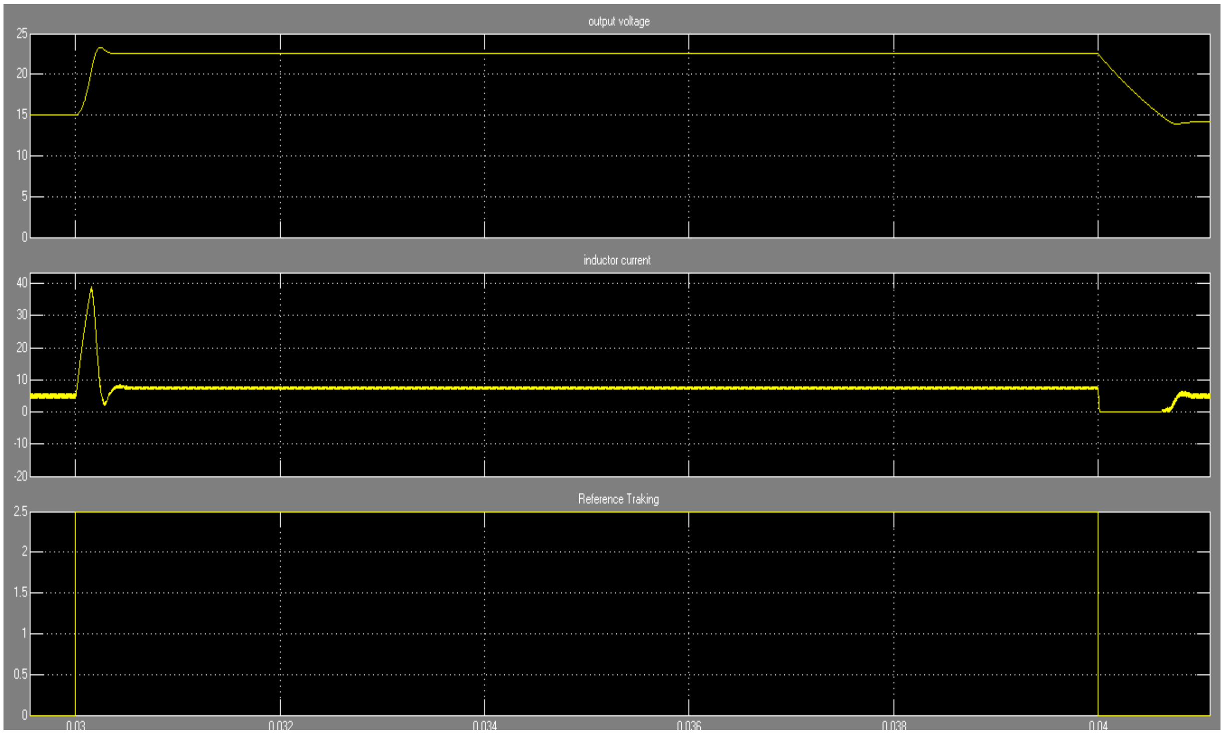


Figure 12: Output Reference Tracking

4.4 Exact Tuning of the PID Controller:

Motivation:

The main motivation to use this paper is that it helps to design the controller on the combination of the phase margin with bandwidth and gain margin with phase margin. It also helps to incorporate the steady state performance. [6]This method also has the advantage of calculating the parameters by use of pen, paper and the calculator.it can be applied to the systems where the plants exact model is not known but a little knowledge of the plants bode response will help us out of the problem.

Objective:

To design a controller that satisfies our requirements of desired frequency response characteristics in software (Matlab) and also in hardware also with a little error and modification.

4.4.1 Method to Tune:

1. First check for the loop transfer function L(s) should satisfy two constraints:

   a) The loop transfer function L(s) should not have any right half plane poles and strictly proper.

   b) The polar plot of the L($j\omega$) for ω≥0 intersects the unit circle and negative semi-real axis only once.

2. When this is done then we can go for the design:

   Standard form of PID controller:

$$C_{PID} = K_P\left(1 + \frac{1}{T_i s} + T_d s\right) \quad \text{........ (1)}$$

   This can be represented in polar form as

$$C_{PID}(j\omega) = M(\omega)e^{j\emptyset(\omega)} \quad \text{...... (2)}$$

   Also plant can be represented as

$$G(j\omega) = |G(j\omega)|e^{j\angle G(j\omega)} \quad \text{........ (3)}$$

3. Now loop transfer function becomes

$$L(j\omega) = |G(j\omega)|M(\omega)e^{j(\emptyset(\omega)+\angle G(j\omega))} \quad \text{..... (4)}$$

4. Now parameters will be calculated as follows:

$$K_P = M_g\cos(\emptyset_g) \quad \text{......... (5)}$$

$$T_i = \frac{tan\emptyset_g + \sqrt{tan^2\emptyset_g + 4\sigma}}{2\omega_g\sigma} \quad \text{....... (6)}$$

$$T_d = T_i\sigma \quad \text{...... (7)}$$

   Here $M_g = M(\omega_g)$

$$\emptyset_g = PM - \pi - \angle G(j\omega)$$

σ = degree of freedom= $\frac{T_d}{T_i}$ this ratio affects the position of zeroes of PID

4.4.2 Verification of the Described Method:

The above is applied to the buck converter to verify whether the parameter values obtained from the previous method gives the satisfactory result or not.

Parameter values of the Buck converter:

Inductor (L) =50μH, Output Capacitor(C) =500μF, R=3Ω, Switching Frequency=100 kHz

Input voltage=28V, Output voltage=15V, Output current=3A

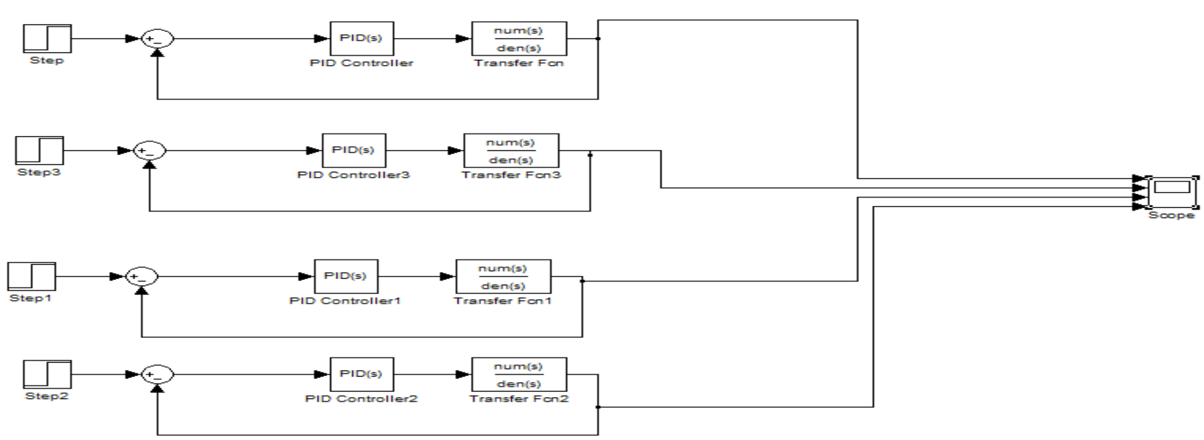
For the above specifications, the control to output transfer function is as follows,

$$G(s) = \frac{v_0}{d} = \frac{2.33}{s^2 \times 2.58 \times 10^{-8} + s \times 16.67 \times 10^{-6} + 1}$$

We choose $\emptyset_d$=52^0 ,$f_c = 5kHz$ and we get controller parameters as follows,

- For $\sigma^{-1} = 5$,$K_P = 6.42$,$T_i = 2.196 \times 10^{-4}$,$T_d = 4.393 \times 10^{-5}$
- For $\sigma^{-1} = 2$, $K_P = 6.42$, $T_i = 9.91 \times 10^{-5}$, $T_d = 4.95 \times 10^{-5}$
- For $\sigma^{-1} = 4$, $K_P = 6.42$, $T_i = 1.798 \times 10^{-4}$, $T_d = 4.5 \times 10^{-5}$

4.5 Results From Exact Tuning Method:



Response of Closed Loop System for Different Values of Degree of Freedom:

Step Response For Different σ^{-1}:

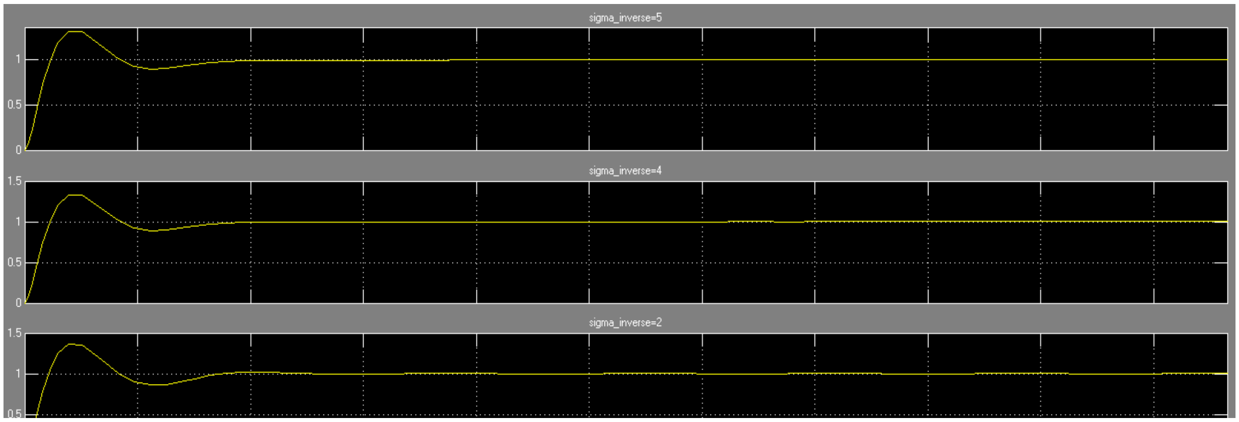


Figure 13: Simulink Model

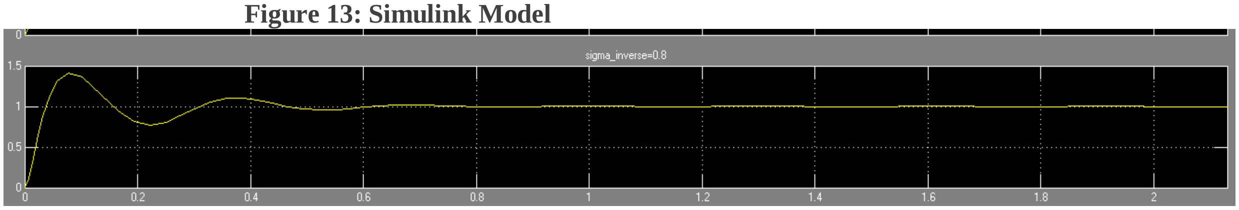


Figure 14: Step Response for different degree of freedom

Inference from the figure:

- As the value of sigma inverse goes on decreasing, the rise time decreases.
- Also the peak overshoots increases.
- Settling time increases and also the oscillations in the output start increasing.
- This is all because the zeroes of the system first real and as the value of sigma inverse decreases they become complex conjugate making system oscillatory.

Bode Plots of System with and without PID Controller:

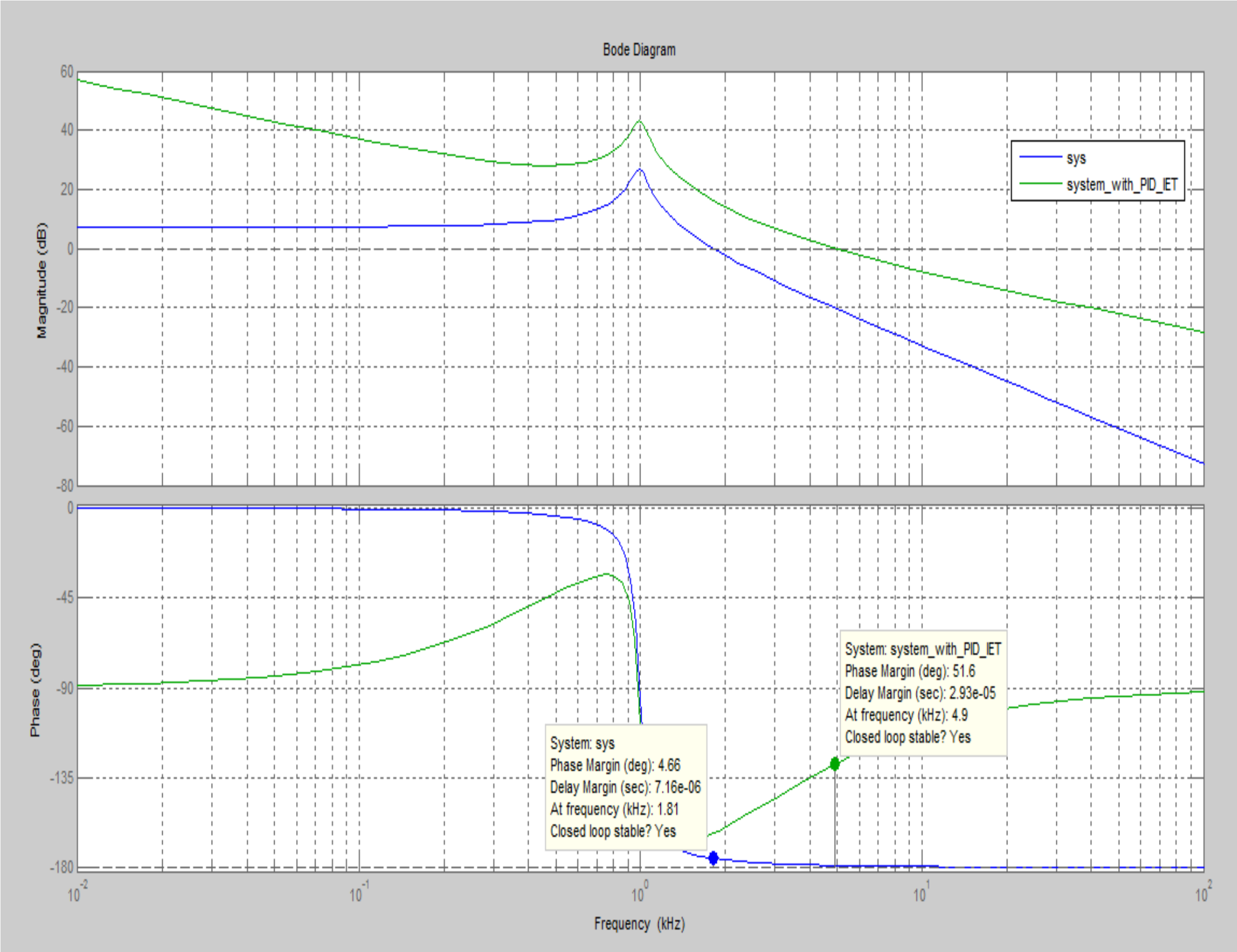


Figure 15: Bode Plots of System with and without PID Controller

Load disturbance rejection:

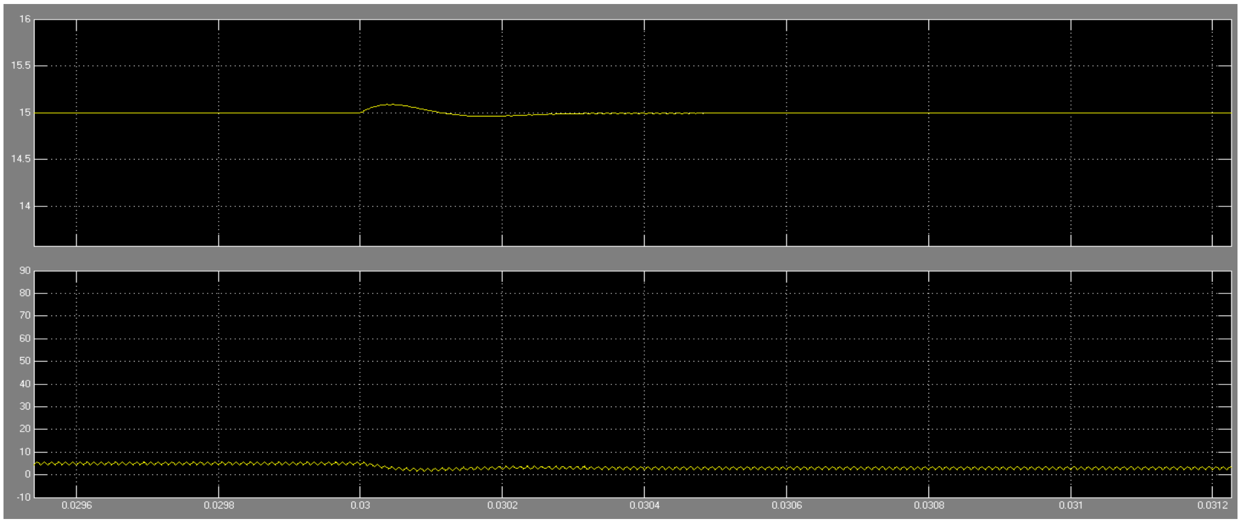


Figure 16: Load disturbance rejection

Reference Voltage Tracking:

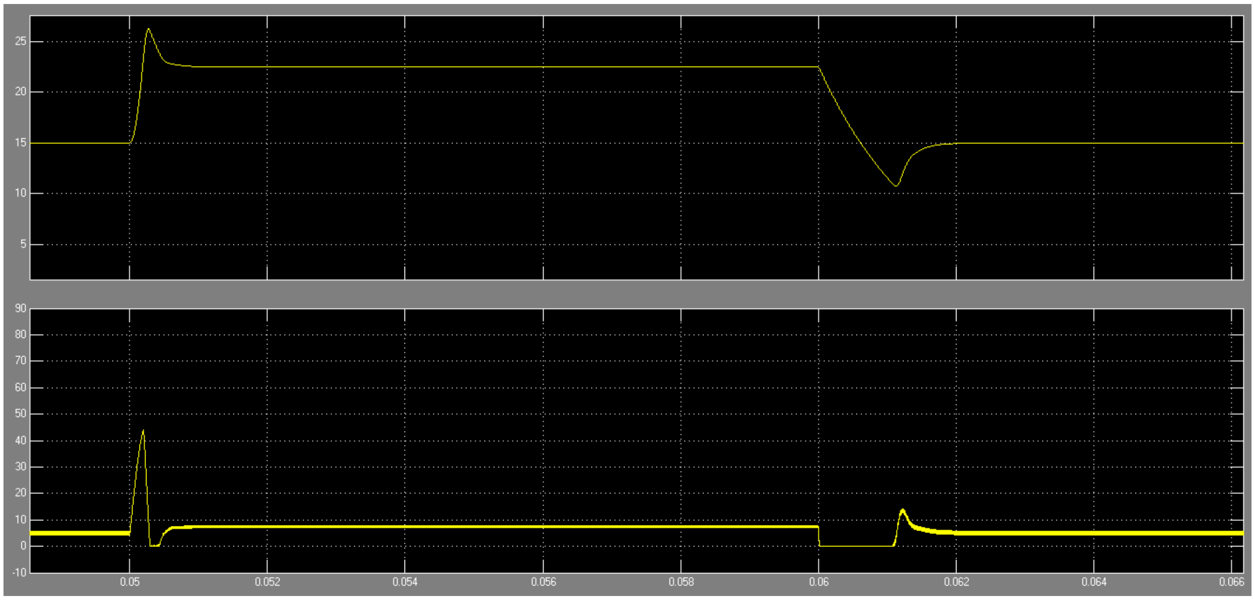


Figure 17: Reference Voltage Tracking

Limitations:

This cannot be applied to systems having right half poles and zeroes as this is the case with boost and the buck-boost converters.

4.6 Type III Compensator Design:

Type-II compensators are widely used in the compensation for DC-DC converters. [7]But it can provide maximum phase boost of 90^0. But if you need the phase boost above 90^0 then you need type-III compensator. There are cases when the phase of power stage converter can reach 180 degree, in this case the type-II compensator cannot stabilize the loop. Then the need for the type-III compensator arises. The extra 90^0 phase boost provided by the type-III compensator helps to achieve higher loop crossover frequency than that achievable by the type-II compensator.

Transfer function of type-III compensator,

$$C(s) = \frac{V_o(s)}{V_i(s)} = \frac{(sC_2R_2 + 1)(sC_3(R_1 + R_3) + 1)}{R_1(C_1 + C_2)s(sC_{12}R_2 + 1)(sR_3C_3 + 1)}$$

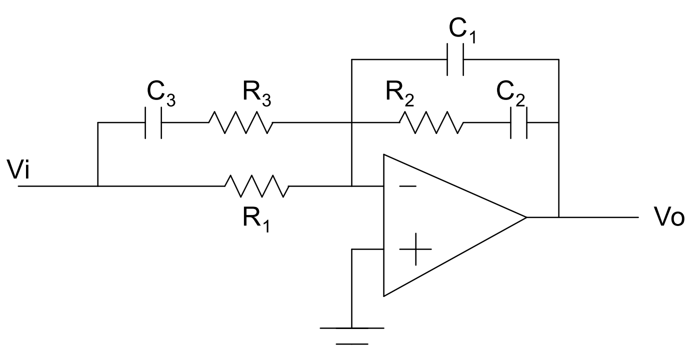


Figure 18: Type-III Structure

4.6.1 Type-III Compensation Network

The type-III compensator has two zeroes and three poles. To get maximum phase at crossover frequency the two zeroes are kept together at frequency lower than the crossover frequency and two poles are kept together at the frequency higher than the crossover frequency, and one pole is kept at the origin to get the low frequency high gain. Therefore the equivalent transfer function will become,

$$C(s) = \frac{K\left(1+\frac{s}{\omega_z}\right)^2}{\left(1+\frac{s}{\omega_p}\right)^2}$$

Where pole and zero frequencies are,

$$\omega_z = \frac{1}{R_2C_2} = \frac{1}{C_3(R_1+R_3)}$$

$$\omega_p = \frac{1}{C_{12}R_2} = \frac{1}{C_3R_3}$$

Where $C_{12} = \frac{C_1C_2}{(C_1+C_2)}$, $K = \frac{1}{R_1(C_1+C_2)}$

4.6.2 Results From Derived Method:

Application to buck converter:

Parameter values of the Buck converter:

Inductor (L) =50µH, Output Capacitor(C) =500µF, R=3Ω, Switching Frequency=100 kHz

Input voltage=28V, Output voltage=15V, Output current=3A

For the above specifications, the control to output transfer function is as follows,

$$G(s) = \frac{v_0}{d} = \frac{2.33}{s^2 \times 2.58\times10^{-8} + s\times16.67\times10^{-6} + 1}$$

We choose $\emptyset_d$=52^0 ,$f_c = 5kHz$ and we get compensator parameters as follows,

R1=5k, R2=9.52k, R3=152, C1=590pF, C2=19.4nF, C3=35.8nF

Frequency Response of the System with and without Compensator

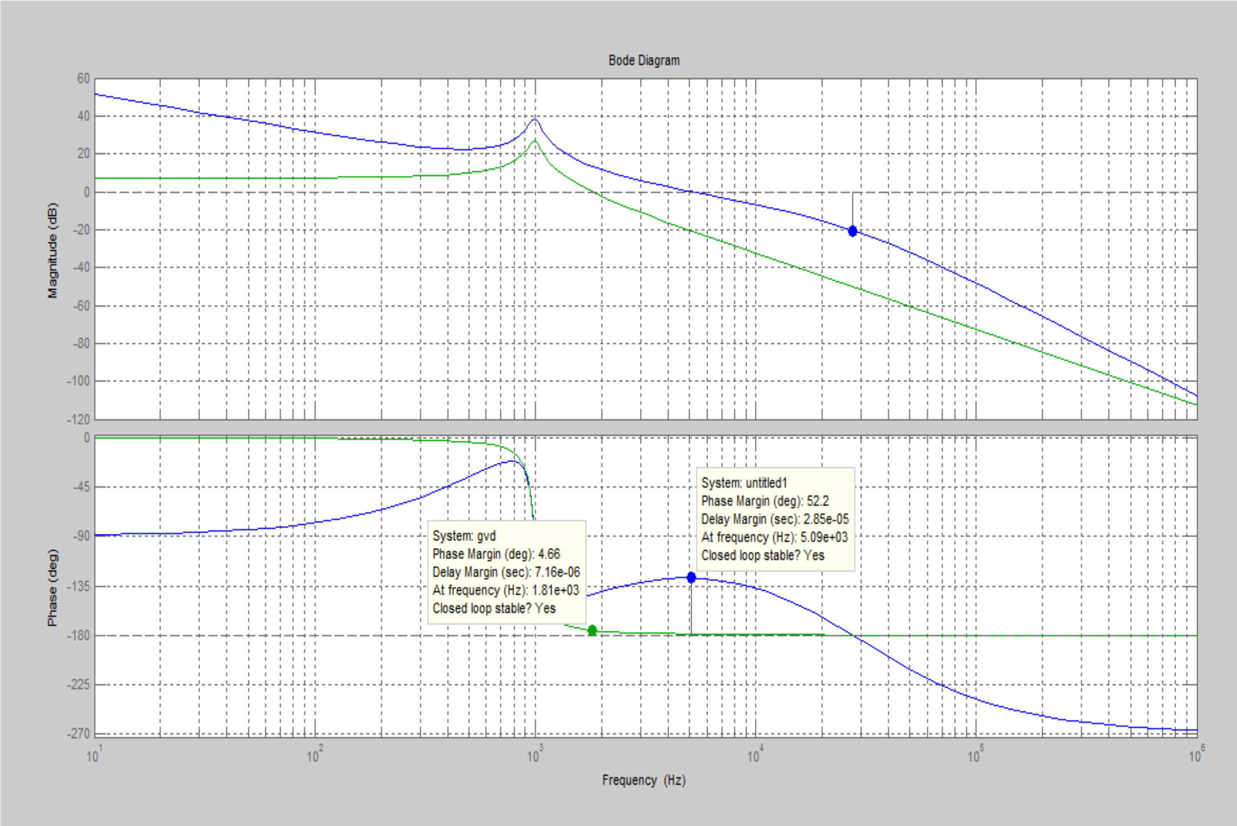


Figure 19: Frequency Response of the System with and without Compensator

4.7 Comparison of Result:

Frequency Domain Characteristics**:**

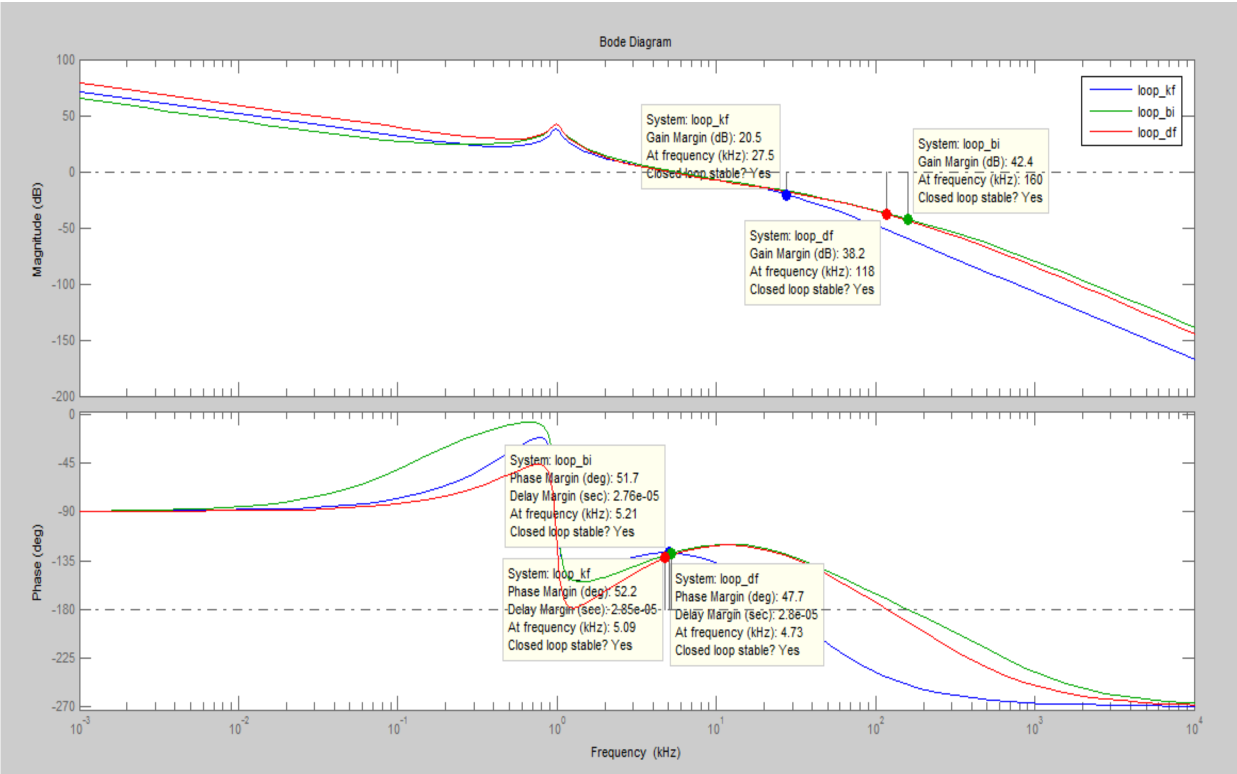


Figure 20: Bode plot of three Compensators together

Table 1: Comparison of Frequency Domain Characteristics

	PM	**Bandwidth**	**GM**	**Gain at 10 Hz**
K Factor	**52.2**	**5.09 KHz**	**20db**	**51.2db**
Degree of Freedom(σ^{-1})	**47.7**	**4.73 KHz**	**38.2db**	**59.1db**
Bode's Integral	**51.9**	**5.21KHz**	**42.2db**	**44.7db**

Step responses of system with compensators**:**

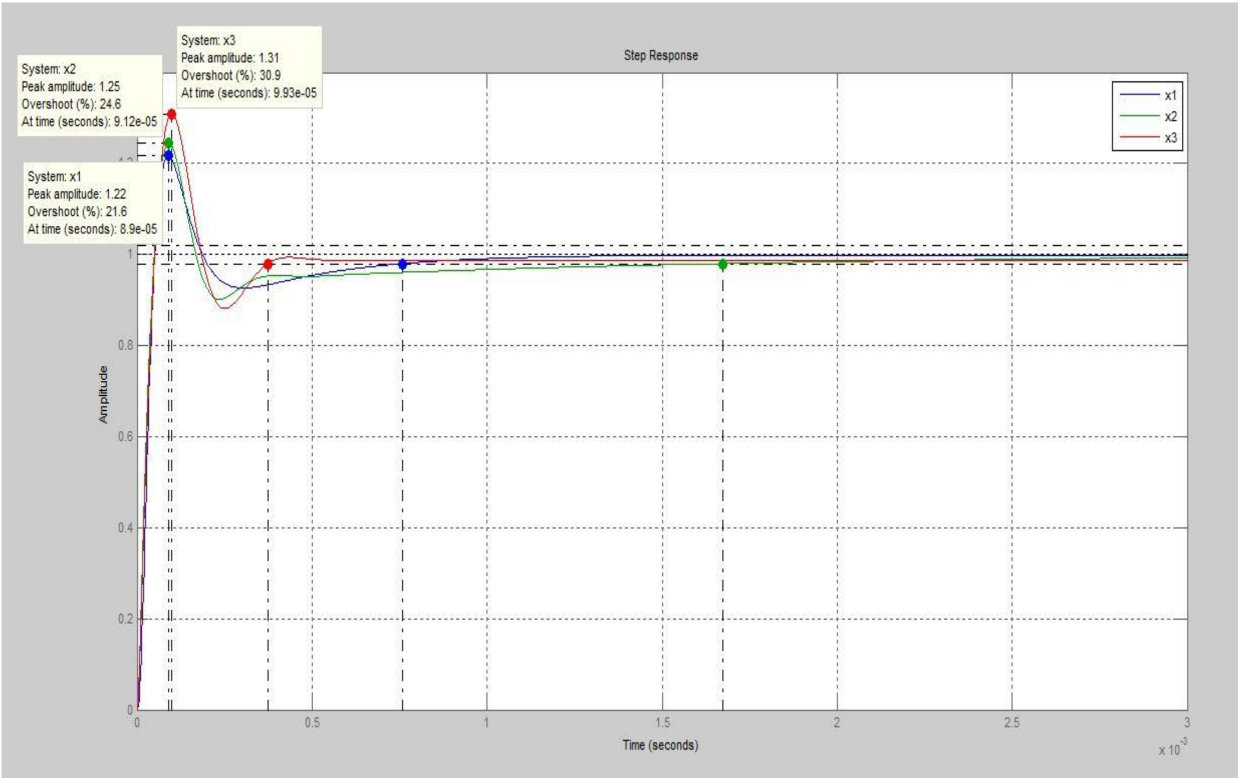


Figure 21: Step responses of system with compensators

Table 2: Comparison of Time Domain Characteristics

	Peak Overshoot	Rise Time	Settling Time
K-Factor	**21.6%**	**33.5µsec**	**758 µsec**
Degree of Freedom(σ^{-1})	**30.9%**	**36.6 µsec**	**374 µsec**
Bode's Integral	**24.5%**	**35.6 µsec**	**167 µsec**

4.8 Experimental Setup:

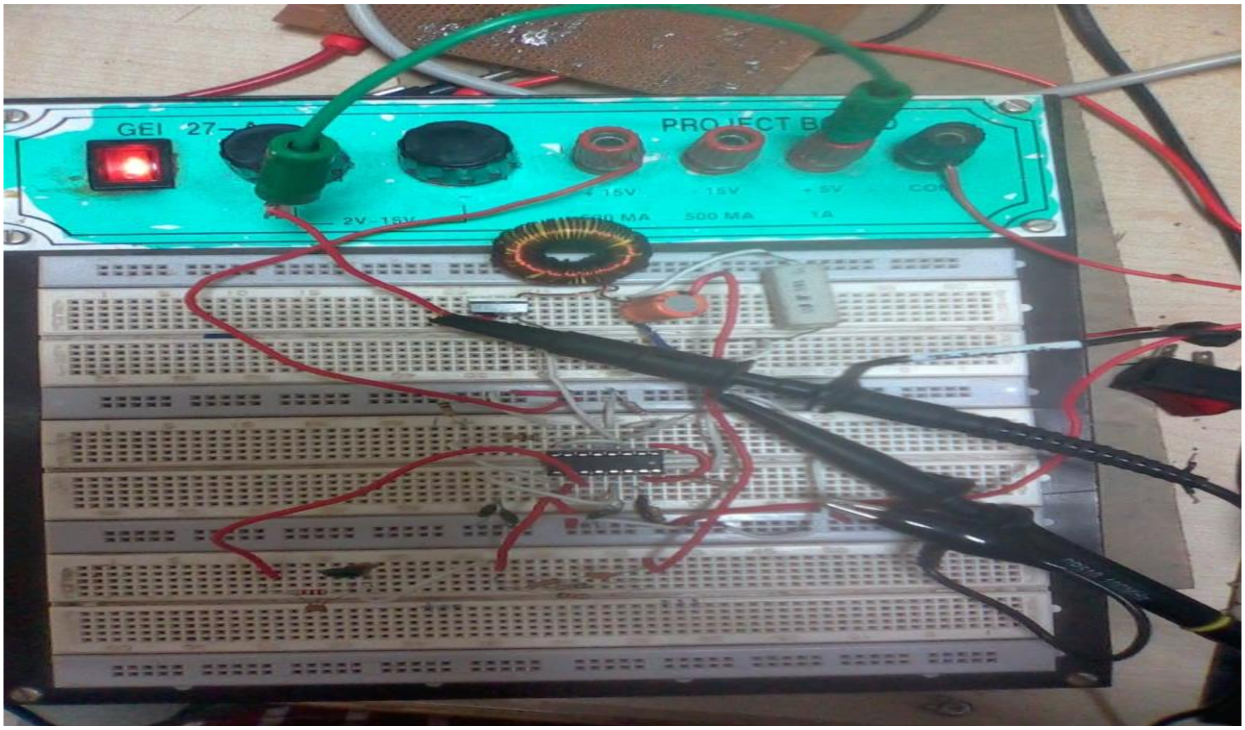


Figure 22: Hardware Image

Experimental Result:

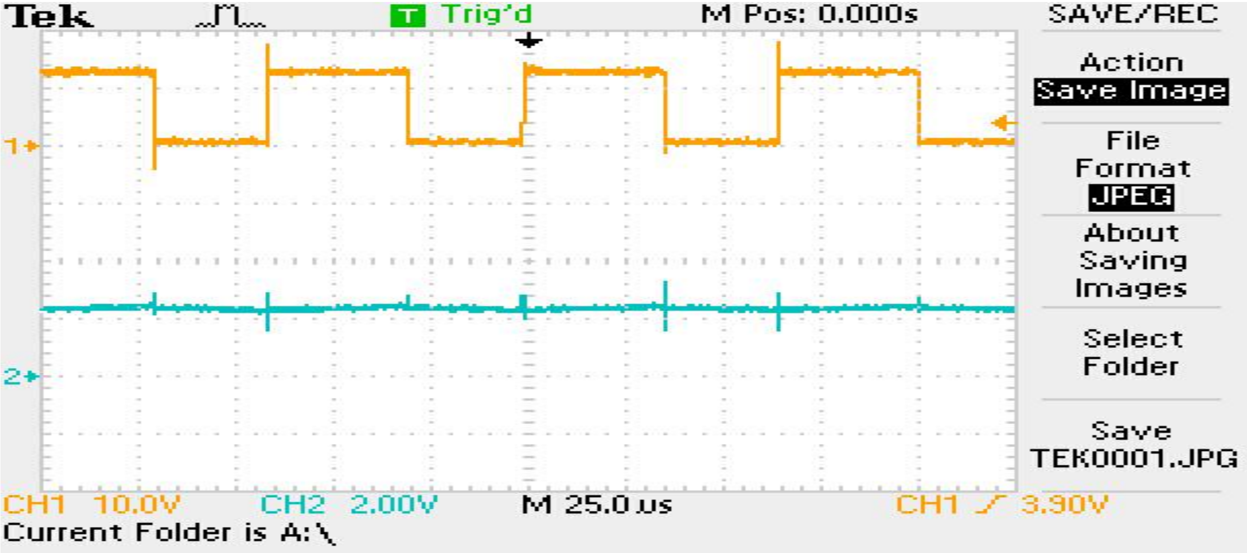


Figure 23: Hardware Result

Conclusion:

The results obtained from these methods are full filling the design criteria and the set parameters are approximately achieved. So to design the PID these two methods are quite easy and give satisfactory results. To compare the methods, the solvability of equation is little bit difficult in Bode's integral method and more number of equations need to be solved compared to exact tuning method. But the results obtained from exact tuning method are best among all these.

References:

[1] H. W. Bode, Network Analysis and Feedback Amplifier Design., New York: Van Nostrand, 945..

[2] K. O. M. C. E. 2. edn., Modern Control Engineering. 2nd edn, Prentice Hall, 2000.

[3] S. Samanta, New methods and Models for Simulation and Compensator Design For Buck Converters in Peak Current Mode Control, Kharagpur: IIT KGP, 2012.

[4] M. K. Kazimierczuk, Pulse-Width Modulated DC-DC power converters, John Wiley and Sons, 2008.

[5] L. a. A. F. Ntogramatzidis, Exact Tuning of PID controllers in control feedback design, IET control theory & applications 5.4, 2011.

[6] R. W. a. D. M. Erickson, Fundamentals of Power Electronics, Springer, 2001.

[7] A. D. G. a. R. L. Karimi, "PID Controllers Tuning in control feedback design," in *Control System Technolgy, IEEE Transactions*, 2003.

[8] L. a. A. P. P. Cao, " "Design Type III Compensator for Power Converters."," in *Power Electronics Technology*, 2011.

[9] K. J. a. R. M. M. Aström, Feedback systems: an introduction for scientists and engineers., Princeton university press, 2010..